Power mean fluid substitution without mineral properties

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Summary

The well-known Voigt upper bound and Reuss lower bound are subcategories of weighted power mean with a power parameter of 1 and -1, respectively. The power mean is a monotonic function of the power parameter. Therefore, the elastic properties of a rock can always be modeled by the power mean with a power parameter in the range of [-1, 1]. The power parameter is pore fluid dependent when the power mean is used to model the effective elastic properties of a porous rock. The process of fluid substitution is to determine the variation of the power parameter. Based on Gassmann equation and regression analysis on laboratory measurement data, an empirical relation is built up to describe the variation of the power parameter with pore fluids. The power parameter for a fully brine-saturated sandstone is well correlated with P-wave velocity and porosity. From the power mean of the P-wave moduli of the mineral matrix and the pore fluids, we derived a fluid substitution procedure that does not need information of the elastic properties of the mineral matrix and the dry rock. We call this procedure power mean fluid substitution. Testing on the laboratory data shows its potential application.

Introduction

The Gassmann theory is well studied because it is one of the most important tools to model the seismic attribute responses of the reservoir rocks due to pore fluid variations. Although these studies are very beneficial for deepening our understanding of the effect of pore fluids on elastic properties of the reservoir rocks, the inclusion of complexity will involve more inputting parameters that are extremely difficult, if not impossible, to attain in the practical applications on the field data. Considering that shear velocity is usually difficult to obtain, Mavko et al. (1995) brought up a P-wave modulus version of the Gassmann equation. It is an approximation and works well for porous rocks, but it still needs input of the P-wave modulus of the mineral matrix, which is often unknown in practical applications and may bring significant uncertainties if an arbitrary value is used.

In this study, we first introduced weighted power mean as a rock physics tool to model the effective properties of porous rocks. Then we studied the influence of the pore fluids on the power parameter of the power mean based on laboratory velocity measurements of reservoir rocks, and developed a practical fluid substitution methodology that can be applied on field data without inputting of the elastic properties of the mineral matrix and dry rock.

Weighted power mean and the bounds

Given a set of values (M_1, M_2, …, M_N), and the corresponding weights (f_1, f_2, …, f_N, and with $\sum_{n=1}^{N} f_n = 1$), the weighted power mean (Bullen, 2003) can be written as

$$M = (f_1 {M_1}^a + f_1 {M_1}^a + \cdots + f_N {M_N}^a)^{1/a}, \qquad (1)$$

Here a is a power parameter. A power mean is also known as a general mean, Hölder mean, or mean of degree. Some well-known means are special cases of the general mean when a takes special values. When a takes values of -1, 0, 1, and 2, the general mean becomes harmonic mean, geometric mean, arithmetic mean, and root-mean-square, respectively. One important feature of the weighted power mean is that it is a monotonic function of the power parameter a for a given set of unequal real numbers, M_1, M_2, …, M_N (Witkowski, 2004). At a=0, a removable discontinuity, the geometric mean takes a different form than the general mean,

$$M = {M_1}^{f_1} {M_2}^{f_2} \ldots {M_N}^{f_N} = \prod_{n=1}^{N} {M_n}^{f_n}. \qquad (2)$$

The specialization of the geometric mean from the general mean is not straightforward and the proof can be found from Rosezweig (2003).

In studying the effective elastic properties of a reservoir rock, the variable M in equations (1) and (2) can be bulk modulus (K), shear modulus (G), P-wave modulus (M), and et al., and f_i is the volume fraction of the constituent phase of a reservoir rock. By such definitions, the weighted harmonic mean is the well-known Reuss bound, and the weighted arithmetic mean is Voigt bound. This point is self-evident when only two phases, the solid frame and pore fluid, are considered:

$$M_{wet} = \left((1-\phi){M_m}^a + \phi {M_f}^a\right)^{1/a}, \qquad (3)$$

where ϕ is the porosity, M_m and M_f are P-wave moduli of the solid frame and the pore fluid, respectively. M_{wet} is

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the effective bulk modulus of the saturated rock. For a=0, equation (3) becomes

$$M_{wet} = {M_m}^{1-\phi} {M_f}^{\phi}. \quad (4)$$

Rock physics data are often plotted with Voigt bound and Reuss bound, or the Hashin-Shtrikman bounds (Hashin and Shtrikman, 1962). Inside the bounds, curves with equal vertical divisions between the bounds are plotted to mark the relative positions of the data points. Fabricius (2003) call these type of curves iso-frame curves. Similar curves can be built using equation (3) by letting a vary from -1 to 1. We call them iso-power curves because each curve represents a power parameter of the same value. Figure 1 illustrates the relationship between the rock physic data in the form P-wave modulus and the iso-power curves. The parameters used to plot the iso-power curves are: Mm=95.7 GPa, Mw=2.3 GPa. The rock physics data are from the laboratory measurements by Han (1986). The short linear features of data points with minute porosity variation are caused by repeated measurements on the same sample at different pressure conditions. The trending of the iso-power curves is generally congruous with the data trend, especially when rocks with different type of lithology are considered separately and the measurements are conducted at the same pressure conditions.

The power parameter a should depend on the rock texture, diagenesis and stress history, and vary within the range of [-1, 1]. For a reservoir rock, if the elastic properties of the mineral matrix, the porosity and the pore fluids are known, and then the power parameter determines its relative position in the Voigt-Reuss bounds, i.e., the relative stiffness of the rock. Therefore, the power mean with the power parameter in the range of [-1, 1] can be utilized to model the elastic properties any rocks.

Dependence of the power parameter on the pore fluids

The power parameter a for a dry rock is in the range of [0, 1] and it is in the range of [-1, 1] for the saturated rock. It is rational to believe that the power parameter will change with pore fluids. If the power parameter for the fully brine-saturated rocks are known, it may be useful to know how the power parameter will change when part of the brine is replaced by oil or gas. If sufficient fund are available, it is possible to build up an empirical relation describing the variation of the power parameter with pore fluids based on a large quantity of laboratory measurements on partially saturated rocks. Instead, our study is based on regular laboratory ultrasonic measurements on fully brine-saturated rocks, and then

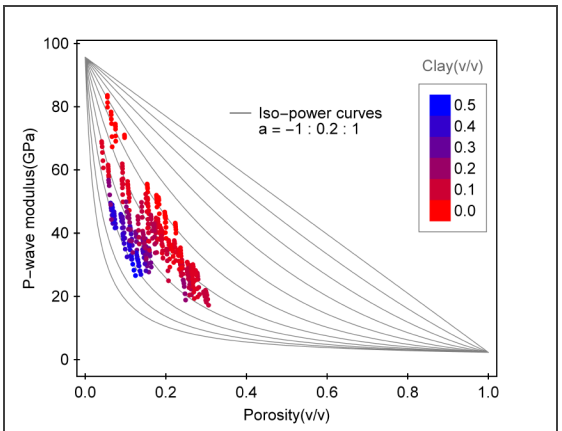

Figure 1: Relationship between Han's data (Han, 1986) and the iso-power curves. a = -1:0.2:1 denotes that the power parameter a increases from -1 to 1 by a step of 0.2. The top curve is the Voigt upper bound, and the bottom curve is the Reuss lower bound.

Gassmann equation in the following form is used to predict the elastic properties of partially saturated rocks:

$$\frac{K_{wet}}{K_m - K_{wet}} - \frac{K_w}{\phi(K_m - K_w)} = \frac{K_{wetp}}{K_m - K_{wetp}} - \frac{K_{fp}}{\phi(K_m - K_{fp})}, \quad (5)$$

where subscript "p" denotes properties related to partial saturation, and K_w is the bulk modulus of the brine. The shear modulus is assumed independent of pore fluids. Finally, equation (3) is used to invert the power parameter for partially saturated rocks, which is denoted as a_{wetp}. Figure 2 shows the cross-plots between the power parameter a_{wet} for fully brine-saturated sandstones and the power parameter a_{wetp} for the same sandstone samples whose pore fluid brine are partially replaced. The ultrasonic measurements on fully saturated sandstones are from Han (1986). The power parameters for partially brine-saturated rocks are well correlated to the power parameters for fully brine-saturated rocks. Based on regression analysis of the data shown in Figure 2, a semi-empirical relation between the power parameter for partially brine-saturated rocks and the power parameter for fully brine-saturated rocks is given as

$$a_{wetp} = 0.274 - 0.275\sqrt{\frac{K_{fp}}{K_w}} + \left(0.435 + 0.600\sqrt{\frac{K_{fp}}{K_w}}\right) a_{wet}. \quad (6)$$

The regression coefficient R^2 for the above formula is 0.994, and the root mean square error is 0.012. In the subsurface conditions, a complete fluid replacement can rarely happen because of the irreducible water and

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complicated pore geometry (Yan et al., 2014; Yan and Han, 2016). Using equations (3) and (6), we can conduct fluid substitution from fully brine-saturated state to a partially saturated state. Same as Gassmann equation, this procedure needs information of the elastic properties of the mineral matrix. This application may be not important because Gassmann equation is sufficient to solve the problem. In the subsurface conditions, often we do not have information about the mineral composition of the reservoirs. It would be very useful if we could develop a procedure of fluid substitution without the mineral matrix properties.

Estimation of pore stiffness and static dry bulk modulus

For a rock at saturation state 1, from equation (3), the P-wave modulus of the mineral matrix can be solved,

$$M_m = \left(\frac{{M_{wet1}}^{a_1} - \phi\, {M_{f1}}^{a_1}}{1-\phi}\right)^{1/a_1}. \qquad (7)$$

Similarly, for the same rock at saturation state 2, we have

$$M_m = \left(\frac{{M_{wet2}}^{a_2} - \phi\, {M_{f2}}^{a_2}}{1-\phi}\right)^{1/a_2}, \qquad (8)$$

From equations (7) and (8), if the P-wave modulus at saturation state 1 is known, then the P-wave modulus at saturation state 2 can be explicitly expressed as

$$M_{wet2} = \left((1-\phi)^{1-\frac{a_2}{a_1}}\left({M_{wet1}}^{a_1} - \phi\, {M_{f1}}^{a_1}\right)^{1/a_1} + \phi\, {M_{f2}}^{a_2}\right)^{1/a_2}. \qquad (9)$$

Based on regression analysis of core measurement data, we can set up a regression formula to estimate the power parameter for full brine-saturated rocks from variables that are also attainable from the well log data. Figure 3 shows the empirical relation of a_{wet} with V_P and porosity (ϕ) based on Han's data (Han, 1986),

$$a_{wet} = -1.689 + 0.256 V_{Pwet} + 1.225\phi^{0.5}. \qquad (10)$$

The correlation coefficient R^2 is 0.711. The root mean square error in estimating a_{wet} is 0.066.

If we assume one of the saturation states is full brine-saturation and the power parameters for fully brine-saturated rocks are known, then the empirical relations (6) and (10) can be combined with equation (9) to conduct fluid substitution on well logs without knowledge of the mineral matrix elastic properties. We call it power mean fluid substitution because the procedure of fluid substitution is primarily to determine the variation of the

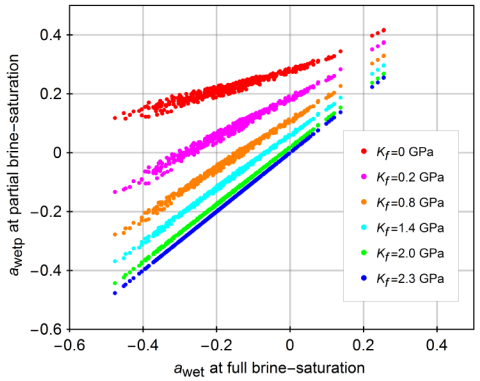

Figure 2: Cross-plots between the power parameters for fully brine-saturated rocks and the power parameters for the same rocks with part of the brine replaced by other fluids.

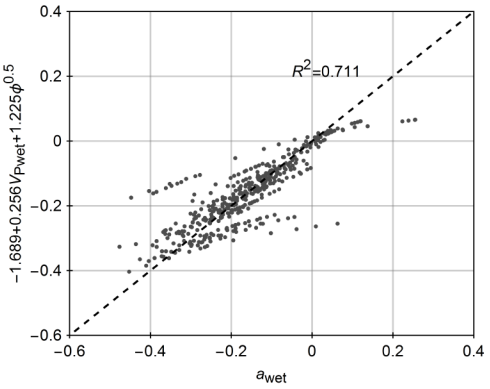

Figure 3: Regression analysis of the power parameter a_{wet} as function of V_{Pwet} and porosity for sandstones. Han's data (Han, 1986) is used.

power parameter of the power mean. If the original state is not full brine-saturation, more algorithmic manipulation is needed to apply power mean fluid substitution. We will discuss it in another paper. Figure 4 shows a comparison of the power mean fluid substitution and Gassmann fluid substitution when the bulk modulus of the pore fluid changes from 2.3 GPa to 0.1 GPa and from 2.3 GPa to 1.4 GPa, respectively. Han's data (Han, 1986) is used. The first fluid substitution scenario may correspond to brine replaced by gas, and the second scenario may correspond to brine replaced by light oil. There should always be irreducible water in reservoir rocks under subsurface conditions.

Using the empirical relations based on regression analysis of the sandstone data by Han (1986), we also applied power mean fluid substitution on carbonate rocks and compared the results with Gassmann fluid substitution, as shown in Figure

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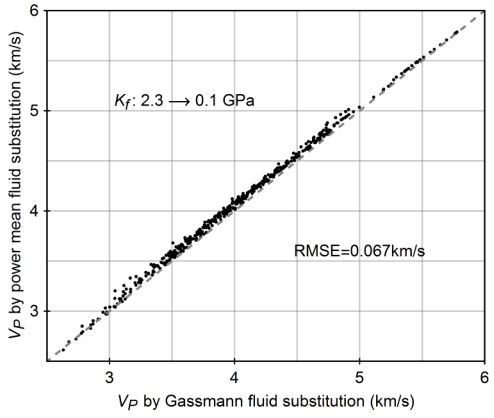

Figure 4: A comparison of Gassmann fluid substitution and power mean fluid substitution. Han's sandstone data (Han, 1986) are used. The empirical relation in Figure 2 is used in the power mean fluid substitution.

5. The carbonate data come from Rafavich et al. (1984), Assefa et al. (2003), Verwer et al., (2008), and Fabricius et al. (2008). The fluid substitution scenarios are same as shown in Figure 4. The differences in the saturation effects predicted by power mean fluid substitution and Gassmann equation are generally greater than those shown in Figure 4 Still, the results are not too bad compared to the traditional Gassmann fluid substitution. If Gassmann fluid substitution is used as standard, the power mean fluid substitution works better on the carbonate datasets by Verwer et al. (2008) and Fabricius et al. (2008) than the other two data sets. This is due to that the correlation of a_{wet} with V_P and porosity for the carbonate datasets by Verwer et al. (2008) and Fabricius et al. (2008) is closer to the empirical relation based on the sandstone data by Han (1986). The results are encouraging because equations (6) and (10) are based on regression analysis of the sandstone data. A better prediction should be achieved if the empirical relations are calibrated using laboratory measurements on core plugs from a local reservoir.

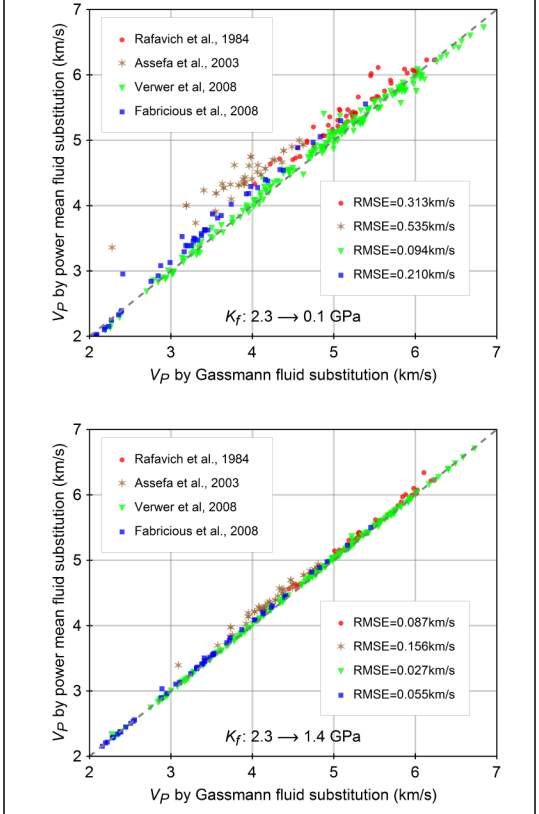

Figure 5: A comparison of Gassmann fluid substitution and the power mean fluid substitution applied on carbonate rocks. The empirical relations derived from the sandstone data are applied on carbonate rocks.

Conclusions

The elastic properties of a rock can always be modeled by weighted power mean with a power parameter in the range of [-1, 1]. Based on weighted power mean and regression analysis of laboratory ultrasonic measurements on core plugs, we brought up the methodology of power mean fluid substitution. It can be applied on field data when elastic properties of the mineral matrix and dry rock are not available.

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