Modeling the effect of pores and cracks interactions on the effective elastic properties of fractured porous rocks

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Summary

We discuss and address two questions for effective medium theories in fractured, porous media: How the pores and cracks interactions could affect the elastic response and seismic anisotropy? And can we physically characterize the pores and cracks interactions? We first use Biot-Gassmann consistency concept to test if an effective medium theory is physically plausible. Then a detailed theoretical analysis and numerical simulations about T-matrix theory for the effective elastic properties will be explored. We also compare the numerical results from different effective medium theories, revealing the physical importance to consider the elastic interactions of pores and cracks.

Introduction

In the past several decades, many theoretical models have emerged to predict the effective elastic properties in fractured, porous media. Most of them are based on strong assumptions with idealizations and simplification of the complexity of real rocks. Most popular approaches to predict the compressibility of a rock containing a finite concentration of pores use non-interaction approximation (NIA) methods to avoid solving pore interactions problems. Those approaches can typically be divided into stiffness based NIA (Eshelby, 1957; O'Connell and Budiansky, 1974; Hudson, 1980) and compliance based NIA (Kachanov et al., 1994; Schoenberg, 1980). But physically they can only work in dilute concentrations of porosity and crack density In order to overcome the dilute limit of NIA, some rock physics schemes, such as differential effective medium (DEM) theory (Nishizawa, 1982; Xu, 1998) and self-consistent (SC) theory (Budiansky, 1965; Berryman, 1995; Hornby et al., 1994), are proposed to handle large concentrations of pores and cracks. It seems that DEM and SC which implicitly simulate the pore interactions can overcome the dilute limit of non-interaction approximation approaches. However, it is not clear that whether those implicit simulations represent the real physical interaction between pores and cracks. Analysis of the assumptions, and resulting characteristics and limitations, of various effective medium theories from the perspective of pores and cracks interactions is presented. The definition and a more physically reliable effective medium theory to characterize the seismic response of the fractured, porous rocks are also described.

Biot-Gassmann Consistency

One approach to verify an effective medium theory is to use Thomsen's (1985) Biot-Gassmann consistency idea. The concept of Biot-Gassmann consistency can be stated as follows: The Biot-Gassmann theory makes only minimal assumptions about the microscopic geometry of the rock. In other words, if the porosity is uniform and the pore pressure can be equilibrated, Biot-Gassmann theory can always work. Therefore, any effective medium theory which does make such assumptions (no pore heterogeneities exist), theoretically, should be a special case of B-G theory (Thomsen, 1985). Biot-Gassmann consistency should be considered as a constraint to test the physical foundation of an effective medium theory. That is to say, if an effective medium theory is physically sound, it should predict the relationship between the elastic response of dry rock and saturated rock as that predicted by Biot-Gassmann theory. It is easy to prove that many non-interacting methods (shape dependent methods, e.g. Eshelby's first order approximation) and bounding methods (shape independent methods, e. g. Voigt-Reuss bound, HS bound) are consistent with Biot-Gassmann predictions.

The requirement of Biot-Gassmann consistency is now tested in the DEM and SC as shown in Figure 1. In our modeling, cracks are vertically aligned and parallel to each other in an isotropic host rock, the resulting cracked rock is transversely isotropic with a horizontal symmetry axis (HTI). The host matrix is assumed to be calcite, the aspect ratio of the cracks is 0.05. There are five independent elements in the effective elastic stiffness tensors, C33 and C11 correspond to P-wave propagating parallel and perpendicular to the crack plane, and C44 and C66 is related to the polarization of S-wave parallel and perpendicular to the crack plane. The volume crack density ε (O' Connell and Budiansky, 1974; Hudson, 1980) is dependent on crack aspect ratio and crack induced porosity. If no other specific instructions, all the numerical simulation in this abstract will be based on this HTI cracked model. Observation of Figure 1 shows that the saturated stiffness C11 by DEM and SC are not in agreement with those predicted by the Brown-Korringa's relations (Figure 1(a) and (b)). This demonstrates that DEM and SC are not consistent with Biot-Gassmann theory, which also implies that the pores and cracks interactions simulated by DEM and SC lack physical foundation.

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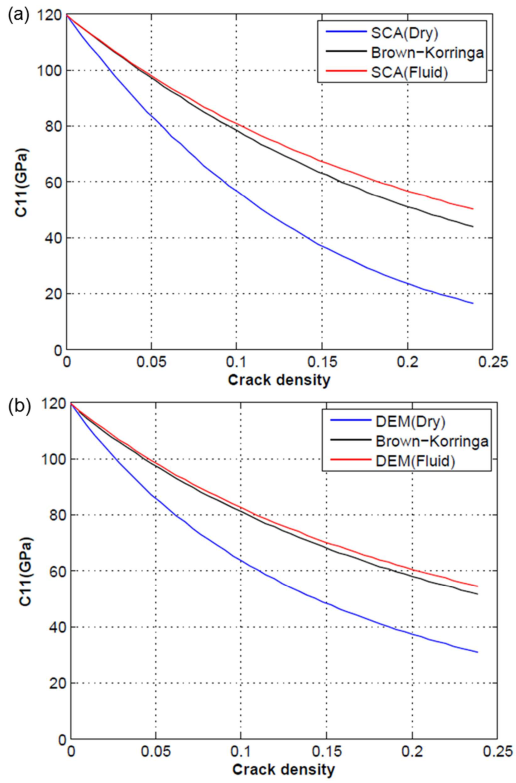

Figure 1: The effective elastic stiffness C11 as a function of crack density simulated by (a) SC, (b) DEM. Blue line represents elastic response of dry rock simulated by DEM and SC. Red line and black line indicate elastic stiffness for saturated rock predicted by DEM and SC and Brown-Korringa relations, respectively.

T-matrix to characterize the elastic interactions

Estimating effective elastic constant of composites can be considered as a many-body problem. One approach to attack such a many-body problem is based on the integral equation or T-matrix approach of quantum scattering theory. This approach takes into account interactions between inclusions based on multiple-point correlation functions. The effective stiffness of the cracked, porous medium is given by Jakobsen et al. (2003a):

$$C_T^{*}=C^{(0)}+\langle \mathrm{T}_1\rangle\left(I-\langle \mathrm{T}_1\rangle^{-1}X\right)^{-1} \qquad (1)$$

Where

$$\langle \mathrm{T}_1\rangle=\sum_{r=1}^{N}v^{(r)}t^{(r)}$$

$$t^{(r)}=\delta C^{(r)}(I-\mathrm{G}^{(r)}\delta C^{(r)})^{-1}$$

$$\delta C^{(r)}=\mathrm{C}^{(r)}-C^{(0)}$$

Here, $\mathrm{G}^{(r)}$ is a fourth-rank tensor given by the strain Green'sfunction integrated over characteristic inclusion shape. $v^{(r)}$ is the volume concentration of inclusion type r, X is the second-order correction for the effects of inclusion tensor. Analytical form of the fourth-rank tensor $\mathrm{G}^{(r)}$ for a transversely isotropic system is given by Mura (1982).

$$X=-\sum_{r=1}^{N}\sum_{s=1}^{N}v^{(r)}t^{(r)}G_d^{(rs)}v^{(s)}t^{(s)}$$

And I is the fourth-rank identity tensor, $G_d^{(rs)}$ represents the two-point interaction between the rth set and sth set of inclusions. The definition of the aspect ratio of inclusion and aspect ratio of spatial distribution are schematically displayed in Figure 2.

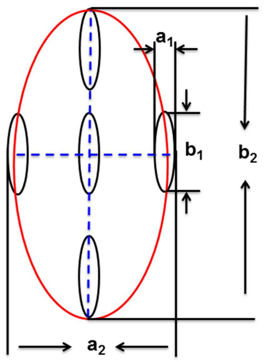

Figure 2: Schematic illustration of a 2D cross section through the 3D ellipsoidal crack distribution in the T-matrix model. The aspect ratio of the individual cracks is a1/ b1, and the aspect ratio of the crack distribution is a2/b2 (After Hu and McMechan, 2009).

Figure 3 is used to illustrate the influence of the aspect ratio of the inclusion and aspect ratio of the spatial distribution on the elastic stiffness of C11. Clearly, the aspect ratio of inclusion has dominant impact on controlling the rock's overall elastic behavior compared with aspect ratio of spatial distribution. It is also interesting to see that stiffness exhibit different sensitivity to the aspect ratio of the spatial distribution when the aspect ratio of inclusion varies. Aspect ratio of spatial distribution generally has bigger impact on the effective elastic stiffness when the aspect ratio of inclusion is lower.

Stress interactions

Figures 3 shows that the computed elastic stiffness decreases with the increasing aspect ratio of spatial distribution. This can be explained by the variation of stress field due to the crack interactions. There are two main crack interaction effects (Kachanov, 1992; Grechka and Kachanov, 2006a; Hu and McMechan, 2009): stress amplification occurs between the tips of cracks while increases the local stress; stress shielding occurs between

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the faces of cracks and decreases the local stress. Normally, stress shielding dominates for stacked cracks, and stress amplification dominates for coplanar cracks (Kachanov, 1992; Grechka and Kachanov, 2006c). Those stress amplification and stress shielding phenomenon are illustrated in Figure 4. Figure 5 presents how the aspect ratio of the spatial distribution affects the elastic stiffness. When the aspect ratio of the spatial distribution decreases, the crack faces will approach closer and closer. So stress shielding will increase stronger than stress amplification, and thus the stiffness will increase accordingly.

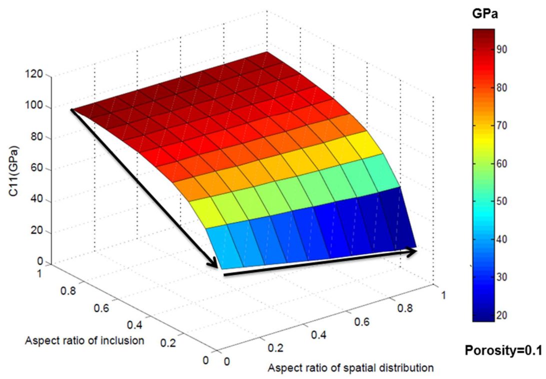

Figure 3: Computed elastic stiffness C11 as a function of aspect ratio of inclusion and aspect ratio of spatial distribution. Porosity is set as 0.1.

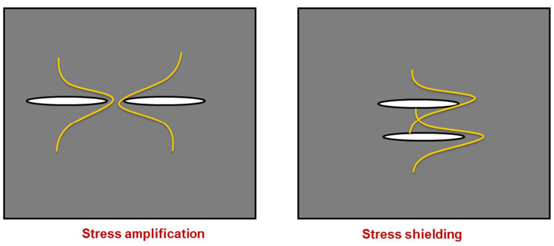

Figure 4: Schematic illustration of stress interaction between two cracks: (left) coplanar cracks, (right) stacked cracks. Yellow lines indicate the iso-stress line.

T-matrix to Biot-Gassmann Consistency

The constraints of Biot-Gassmann consistency are applied to the T-matrix as shown in Figure 6. It turns out that the saturated stiffness simulated by T-matrix exactly matches those predicted by the Brown-Korringa's relations. This suggests that T-matrix is consistent with the Biot-Gassmann theory. However, note that the SC and DEM which implicitly simulate the pore interactions are not consistent with Biot-Gassmann as demonstrated in Figure 1 (a) and (b). On the other hand, this observation verifies that the T-matrix simulate the crack interactions with physical foundation. Figure 7 illustrates the fluid saturation effect in the case where both stiff pores and thin cracks are present. The simulated saturated stiffness by T-matrix tends to deviate from the prediction by Brown-Korringa's relations when cracks occur, which suggest that the T-matrix is not consistent with B-G theory if pore heterogeneities exist.

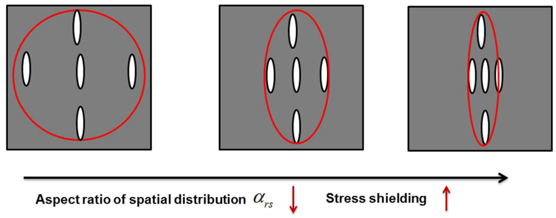

Figure 5: Schematic illustration of how the aspect ratio of spatial distribution affects the stress field.

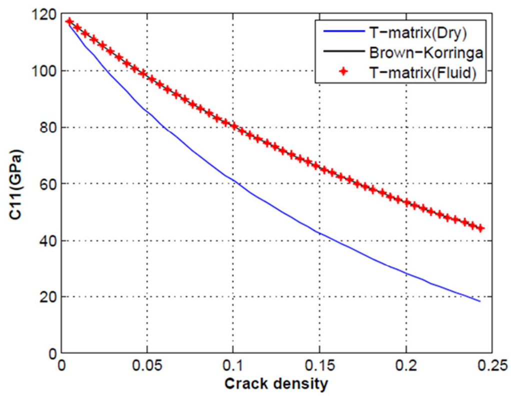

Figure 6: Illustration of T-matrix to Biot-consistency. The effective elastic stiffness C11 is diaplayed as a function of crack density.

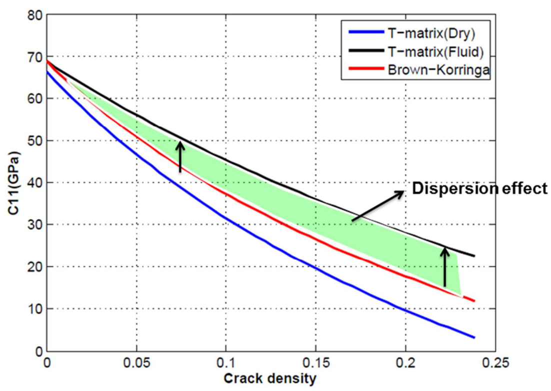

Figure 7: Dispersion effect of the elastic stiffness C11 with increasing crack density. The solid matrix is calcite and the matrix (stiff) porosity is 0.2. The aspect ratio of the matrix porosity and cracks are set as 0.5 and 0.05 respectively.

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Comparison of different effective medium theories

The comparisons of T-matrix with Hudson's crack theory, compliance based NIA, Self-consistent and DEM model are displayed in Figure 8. As expected, the several predictions are close to each other when the crack density is small, but deviate significantly at high crack density. This demonstrates the importance of including the effects of spatial distribution when trying to deal with non-dilute mixtures of highly contrasting material properties. Hudson's crack theory typically breaks down at high crack density. The compliance based NIA gives the best match with the T-matrix when the aspect ratio of spatial distribution is very small, which represents stress shielding dominating the crack interaction effect. However, this should not be considered as physical equivalence, as the physical assumptions of the two effective medium theories are different. The compliance based NIA does not include the crack interaction or the effect of spatial distribution, whereas those are explicitly characterized in the T-matrix formulation. Additional insight can be gained from this comparisons is that the Self-consistent and DEM prediction approach the T-matrix prediction when the aspect ratio of the spatial distribution is 1, and this is in accordance with the assumption of SC and DEM, in which the cracks are distributed randomly.

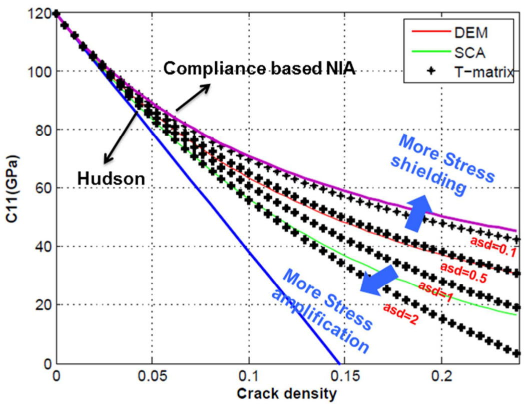

Figure 8: Comparison of C11 as a function of crack density predicted by different effective medium theories.

How the spatial distributions of cracks affect the seismic anisotropy for HTI media (Figure 9) is also examined. It is clear that the impact of spatial distribution on the seismic anisotropy become increasingly important when the inclusion concentrations increase beyond the dilute limit. Generally, the seismic anisotropy will decrease as the aspect ratio of spatial distribution decrease. In other words, the amplitude of seismic anisotropy will get stronger when the stress amplification dominates the crack interactions. Moreover, the gamma parameter, which is a measure of shear wave splitting discussed in many papers (see Bakulin et al., 2000) is close to crack density which indicates the degree of fracturing (Grechka and Kachanov, 2006c). Figure 9 clearly shows that this conclusion is mainly based on Hudson's theory. Nevertheless, T-matrix normally gives a higher prediction about crack density based on the anisotropic parameter gamma, and this effect is more evident when the stress shielding dominates the crack interactions at high crack density.

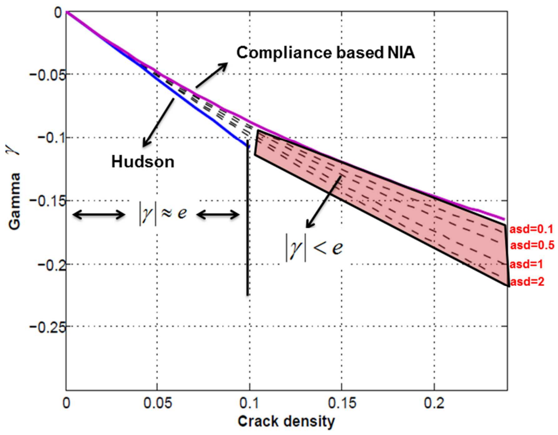

Figure 9: Comparisons of predictions of Thomsen's anisotropic parameter Gamma as a function of crack density. Black dashed lines indicate T-matrix prediction with different aspect ratio of spatial distribution marked.

Conclusions

We have discussed how to select a convincing effective medium theory to characterize the elastic response for fractured, porous rock. A good effective medium theory should satisfy three conditions: first of all, it should work beyond dilute limit; secondly, it should be consistent with Biot-Gassmann theory; finally, it should characterize the pores and cracks interactions with physical foundations. DEM and SC implicitly simulate the elastic interactions between pores and cracks, but are not Biot-Gassmann consistent. Numerical results show that T-matrix can produce physically plausible results even at large concentrations of pores, and is always Biot-Gassmann consistent when no pore heterogeneity exists. This suggests that T-matrix explicitly simulate the pores and crack interactions with physical foundations. We also use the T-matrix theory to study how the spatial distribution of pores and cracks affect rock's elastic response and seismic anisotropy, and this impact cannot be ignored when the inclusion concentrations (porosity or crack density) increases beyond the dilute limit.

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