GEOPHYSICS, VOL. 69, NO. 2 (MARCH-APRIL 2004); P. 398–405, 8 FIGS., 2 TABLES.
10.1190/1.1707059

Gassmann's equation and fluid-saturation effects on seismic velocities

De-hua Han* and Michael L. Batzle‡

ABSTRACT

Gassmann's (1951) equations commonly are used to predict velocity changes resulting from different pore-fluid saturations. However, the input parameters are often crudely estimated, and the resulting estimates of fluid effects can be unrealistic. In rocks, parameters such as porosity, density, and velocity are not independent, and values must be kept consistent and constrained. Otherwise, estimating fluid substitution can result in substantial errors. We recast the Gassmann's relations in terms of a porosity-dependent normalized modulus K_n and the fluid sensitivity in terms of a simplified gain function G. General Voigt-Reuss bounds and critical porosity limits constrain the equations and provide upper and lower bounds of the fluid-saturation effect on bulk modulus. The "D" functions are simplified modulus-porosity relations that are based on empirical porosity-velocity trends. These functions are applicable to fluid-substitution calculations and add important constraints on the results. More importantly, the simplified Gassmann's relations provide better physical insight into the significance of each parameter. The estimated moduli remain physical, the calculations are more stable, and the results are more realistic.

INTRODUCTION

With improved resolution and cost efficiency, seismic technologies have gained a central position in reservoir delineation and monitoring. Rock physics is an essential link connecting seismic data to the presence of in situ hydrocarbons and to reservoir characteristics. Modeling the effects of fluid on rock velocity and density is a basic method used to ascertain the influence of pore fluids on seismic data. Gassmann's (1951) equations are the relations most widely used to calculate seismic-velocity changes resulting from different fluid saturations in reservoirs. These equations predominate in the analysis of direct hydrocarbon indicators (DHI), such as amplitude "bright spots," amplitude variation with offset (AVO), and time-lapse reservoir monitoring.

Despite the popularity of Gassmann's equations and their incorporation in most software packages for seismic reservoir interpretation, important aspects of these equations have not been thoroughly examined. Many of the basic assumptions are invalid for common reservoir rocks and fluids. Previous efforts to understand the operation and application of Gassmann's equations (Han, 1992; Mavko and Mukerji, 1995, Mavko et al., 1998; Sengupta and Mavko, 1999) have focused mostly on individual parameter effects.

Recently, Nolen-Hoeksema (2000) made a detailed effort to quantify changes in the pore-space modulus in response to changes in fluid modulus. He introduced an effective fluid coefficient by differentiating "the bulk modulus of the fluid-filled pore space K_{pore}." He included two ratios to control the effective fluid coefficient: the ratio of fluid modulus to solid grain modulus and the ratio of the Biot coefficient (1941) to porosity. However, the physical meaning of the effective fluid coefficient was not clarified. The fluid effect was in conjunction with other rock parameters. The Voigt and Reuss models were introduced to test the fluid effect. We need to extend his analysis to derive both the mechanical bounds for porous media and the magnitude of the fluid effect.

Because the full implications of parameter interactions in Gassmann's equation are not well understood, in general practice, no constraints are placed on input parameters and there is no quality control of the results. In particular, problems arise in automated analysis in which results are usually taken at face value. In this paper, we briefly list the assumptions for Gassmann's equation. We then derive mechanical bounds for the input parameters that provide stricter constraints on calculated results. The specific physical properties controlling the fluid-saturation effect are then better understood.

Effect of fluid saturation on seismic properties

The seismic response of reservoirs is directly controlled by compressional (P-wave) and shear (S-wave) velocities V_p and V_s respectively along with densities. Figure 1a shows measured

Manuscript received by the Editor September 19, 2001; revised manuscript received July 26, 2003.
*University of Houston, Department of Geoscience, 312 Science & Research Building, Houston, Texas 77204-5007. E-mail: dhan@mail.uh.edu.
‡Colorado School of Mines, Department of Geophysics, Golden, Colorado 80401-1887. E-mail: mbatzle@mines.edu.

dry and water-saturated P- and S-wave velocities of sandstone as a function of differential pressure. With water saturation, P-wave velocity increases slightly, whereas S-wave velocity decreases slightly. However, neither P- nor S-wave velocity is the best indicator of any fluid saturation effect because of the coupling between P- and S-waves through the shear modulus and bulk density:

$$V_p = \sqrt{\frac{K + 4/3 \times \mu}{\rho}} = \sqrt{\frac{M}{\rho}},$$
$$V_s = \sqrt{\frac{\mu}{\rho}}, \quad (1)$$

where K is bulk modulus, μ is shear modulus, M is the compressional modulus, and ρ is the bulk density. In contrast, if we plot bulk and shear moduli as functions of pressure (Figure 1b), the water-saturation effect becomes evident: (1) the bulk modulus increases about 50%, whereas (2) shear modulus remains constant.

Clearly, bulk modulus is more sensitive to water saturation. The bulk-volume deformation produced by a passing seismic wave results in a pore-volume change and causes a pressure increase in pore fluid (water). This pressure increase stiffens the rock frame and causes an increase in bulk modulus. Shear deformation, however, does not produce a pore-volume change, and consequently different fluids do not affect shear modulus. Therefore, any fluid-saturation effect should correlate mainly to a change in bulk modulus.

Gassmann's equation

Gassmann's equations provide a simple model for estimating the fluid-saturation effect on bulk modulus. We prefer the following Gassmann's formulation because of its clear physical meaning:

$$K_s = K_d + \Delta K_d$$
$$\Delta K_d = \frac{K_0(1 - K_d/K_0)^2}{1 - \phi - K_d/K_0 + \phi \times K_0/K_f}, \quad (2)$$
$$\mu_s = \mu_d, \quad (3)$$

where K_0, K_f, K_d, K_s, are the bulk moduli of the mineral grain, fluid, dry rock, and saturated rock frame, respectively; ϕ is porosity; and μ_s and μ_d are the saturated and dry-rock shear moduli. ΔK_d is an increment of bulk modulus as a result of fluid saturation of dry rock. These equations indicate that fluid in pores will affect bulk modulus but not shear modulus, which is consistent with our earlier discussion. As Berryman (1999) pointed out, a shear modulus that is independent of fluid saturation results directly from the assumptions used to derive Gassmann's equation.

Numerous assumptions are involved in the derivation and application of Gassmann's equation:

1. the porous material is isotropic, elastic, monomineralic, and homogeneous;
2. the pore space is well connected and in pressure equilibrium (zero-frequency limit);
3. the medium is a closed system with no pore-fluid movement across boundaries;
4. there is no chemical interaction between fluids and rock frame (shear modulus remains constant).

Many of these assumptions may not be valid for hydrocarbon reservoirs and depend on rock and fluid properties and the in-situ conditions. For example, most rocks are anisotropic to some degree, invalidating assumption (1). The work of Brown and Korringa (1975) provides an explicit form for an

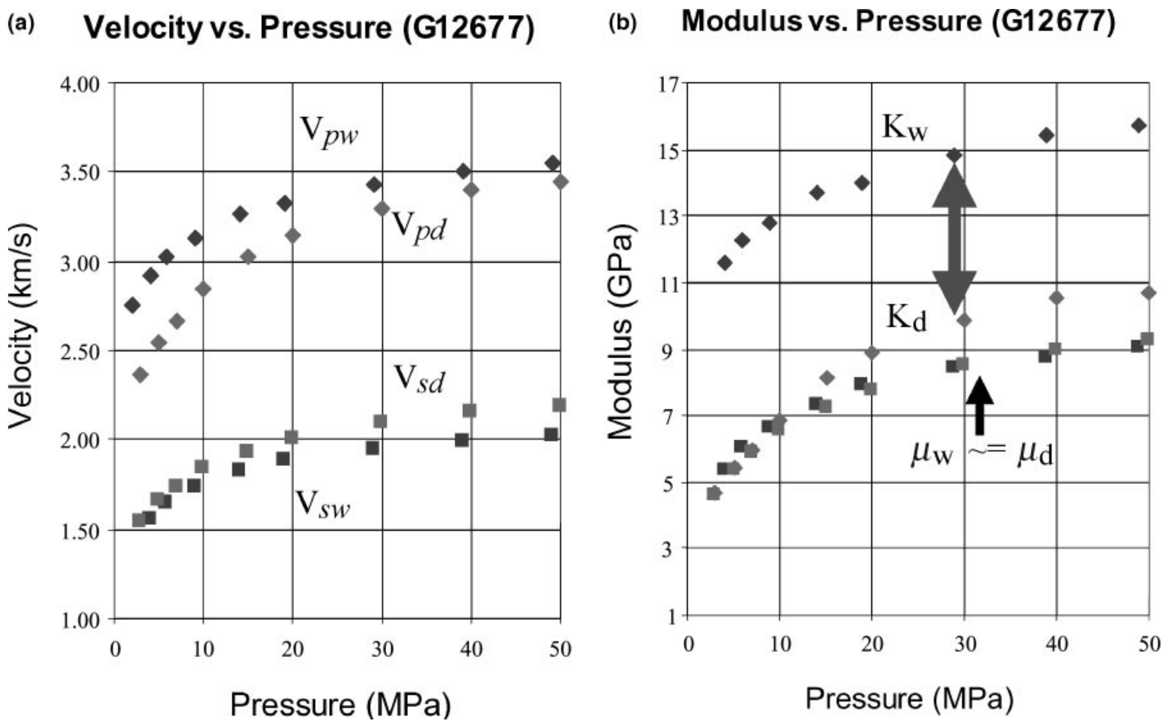

Figure 1. (a) Measured P- and S-wave velocities on a sandstone sample at dry and water-saturated states, as a function of pressure. (b) Bulk and shear moduli at dry and water-saturated states, as a function of pressure.

anisotropic fluid substitution. Also, in seismic applications, it is normally assumed that Gassmann's equation works best for seismic data at frequencies less than 100 Hz (Mavko et al., 1998). Recently published laboratory data (Batzle et al., 2001) show that acoustic waves may be dispersive in rocks within the typical seismic band, invalidating assumption (2). In such cases, seismic frequencies may still be too high for the application of Gassmann's equation. Pore pressures may not have enough time to reach equilibrium, and the rock remains unrelaxed or only partially relaxed. A Gassmann-type calculation provides an estimate of relaxed velocity at zero frequency, which is a lower bound of the fluid-saturation effect. Other similar violations of the assumptions often lead to the misapplication of Gassmann's equation. We will discuss this issue in detail in a separate paper.

SIMPLIFIED GASSMANN'S EQUATION

Gassmann's formulation is straightforward, and the simple input parameters typically can be directly measured or assumed. This simplicity is a primary reason for its wide application in geophysical techniques. However, derivation of the rock and fluid input parameters frequently remains ambiguous. As a consequence, we frequently have little control over the validity of fluid-substitution calculations. We have made efforts to clarify how the fluid-saturation effects are controlled by rock parameters (Han, 1992). A simple graphic construction developed by Mavko and Mukerji (1995) introduces an "intercept" porosity φ_R for a dry rock that is based on bulk porosity, dry-bulk modulus, and solid-frame modulus. However, the physical meaning of the intercept porosity is not clear. Additionally, multiple-parameter effects on Gassmann's calculation remain ambiguous. As a consequence, for application of Gassmann's calculation, generally we have not been able either to constrain input parameters or to obtain quality control on the results. In this section, we regroup Gassmann's equation with combined rock parameters. Under certain conditions, we can simplify this equation further and clearly define the controlling parameters for fluid-saturation effects.

The primary measure of a rock's velocity sensitivity to fluid saturation is its normalized modulus K_n, the ratio of dry bulk modulus to that of the mineral:

$$K_n = K_d/K_0. \tag{4}$$

This function can be complicated and depends on rock texture (porosity, clay content, pore geometry, grain size, grain contact, cementation, mineral composition, and so on) and reservoir conditions (pressure and temperature). This K_n can be determined either empirically or theoretically. For relatively clean sandstone at high differential pressure (>20 MPa), the complex dependence of $K_n(x, y, z, \ldots)$ can be simplified as a function of porosity:

$$K_n(x, y, z, \ldots) \cong K_n(\phi). \tag{5}$$

From equation (2), the bulk-modulus increment ΔK_d is then equal to

$$\Delta K_d = \frac{K_0 \times [1 - K_n(\phi)]^2}{1 - \phi - K_n(\phi) + \phi \times K_0/K_f} \tag{6}$$

where $(1 - K_n(\phi))$ is the Biot parameter α (Biot, 1941). The Biot parameter α is a relative measure of the difference between the mineral and dry-frame moduli. Furthermore, if we apply the Voigt bound for K_n, and because usually $K_0 \gg K_f$, it is reasonable to assume that

$$0 \leq 1 - \phi - K_n(\phi) \ll \phi \times K_0/K_f \tag{7}$$

for sedimentary rocks with high porosities ($\phi > 15\%$). Therefore, the fluid-saturation effect of the Gassmann equation can be simplified as

$$K_s = K_d + \Delta K_d \approx K_d + G(\phi) \times K_f, \tag{8}$$

where $G(\phi)$ is the simplified gain function defined as

$$G(\phi) = \frac{[1 - K_n(\phi)]^2}{\phi} = \frac{\alpha^2}{\phi}. \tag{9}$$

Equation (8) is a simplified form of Gassmann's equation, with clear physical meaning: fluid-saturation effects on the bulk modulus are proportional to a simplified gain function $G(\phi)$ and the fluid modulus K_f. The $G(\phi)$ in turn depends directly on dry-rock properties: the normalized modulus and porosity. In general, $G(\phi)$ is independent of fluid properties (ignoring interactions between rock frame and pore fluid). Equation (9) also shows that the normalized modulus or the Biot parameter must be compatible with porosity. Otherwise, $G(\phi)$ can be unstable, particularly at small porosities. We need to know both the simplified gain function of the dry rock frame and the pore-fluid modulus to evaluate the fluid-saturation effect on seismic properties.

Figure 2 shows that for sandstone samples (Han, 1986) at high differential pressure (>20 MPa), the K_s of water-saturated sands calculated using the simplified form is overestimated by 3% for porosities greater than 15%. These errors will decrease significantly with a low fluid modulus (gas and light-oil saturation). For low-porosity sands with high clay content, the simplified Gassmann's equation substantially overestimates water-saturation effects.

This simplified Gassmann's equation has a clear physical meaning with systematic errors. This formulation can guide us in applying Gassmann's equation and in assessing the validity of such calculations.

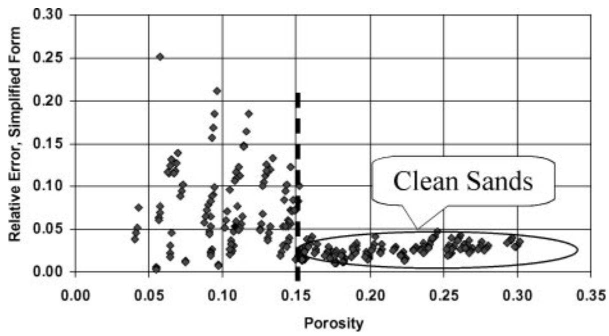

Figure 2. The simplified Gassmann's formulation overestimates the water-saturation effect on bulk modulus of sandstone samples (1986) at a differential pressure greater than 20 MPa. For porous sands (porosity >15%), the error is around 3%.

CONSTRAINTS ON GASSMANN'S EQUATION

The assumptions contained in Gassmann's equation do not constrain basic rock parameters. In equations (2), there are five parameters, and usually the only applied constraint is that the parameters are physically meaningful (>0). When one applies Gassmann's equation, one generally handles input parameters as being completely independent. Values for K_0 and K_f are estimated or assumed. K_s or K_d are calculated from V_p and V_s, and density either comes from log data or is estimated along with ϕ. Incompatible or mismatched data often generate wrong or even unphysical results, such as a negative modulus. Unrealistic results can be hidden when one is performing a fluid modulus substitution without examining K_d. In reality, only K_0 and K_f are completely independent. K_s, K_d, and ϕ are actually closely correlated. Bounds on K_d as a function of ϕ, for example, constrain the bounds of K_s.

The Voigt bound

Assuming the porous medium is a Voigt material, which is a high bound for K_d,

$$K_d = K_0 \times (1 - \phi). \tag{10}$$

Substituting equation (10) into Gassmann's equation (2) gives

$$\Delta K_{d\,\min} = \phi \times K_f, \tag{11}$$

and

$$K_s = K_d + \Delta K_{d\,\min} = K_0 \times (1 - \phi) + \phi \times K_f. \tag{12}$$

Because this Voigt bound is the stiffest upper limit, the fluid-saturation effect on bulk modulus here ($\Delta K_{d\,\min}$) will be a minimum (Figure 3). This is the first constraint derived from the Gassmann's equation: The minimum of bulk modulus increment resulting from fluid saturation is proportional to the porosity of the rock (the simplified gain function $G(\phi) = \phi$) and the modulus of the pore fluid.

We know that fluid modulus is a positive value, but $K_f \ll K_0$. Therefore, from equations (10) and (11),

$$K_0 \geq K_s \geq K_d. \tag{13}$$

The Reuss bound

The low modulus bound for porous media is the Reuss bound:

$$\frac{1}{K_R} = \frac{(1 - \phi)}{K_0} + \frac{\phi}{K_f}, \tag{14}$$

$$K_R = \frac{K_0 \times K_f}{(1 - \phi) \times K_f + \phi \times K_0}. \tag{15}$$

For completely empty (dry) rocks, the fluid modulus K_f is equal to zero. Thus, both the Reuss bound and the normalized modulus (K_{nR}) for a dry rock in this case equals zero (for nonzero porosity):

$$K_{nR}(\phi) = K_d / K_0 = 0. \tag{16}$$

Substituting equation (16) into the Gassmann's equation (2), we find the fluid-saturation effect on bulk modulus when the frame is at this lower bound:

$$\Delta K_{d\,\max} = \frac{K_0}{1 - \phi + \phi \times K_0 / K_f} = K_R. \tag{17}$$

Note that the modulus increment ΔK from dry to fluid saturation is equal to the Reuss bound (equation 15):

$$K_s = K_d + \Delta K_{d\,\max} = K_R. \tag{18}$$

Again, Gassmann's equation is consistent with the dry and fluid-saturated Reuss bounds. The $\Delta K_{d\,\max}$ is the maximum fluid-saturation effect predicted from Gassmann's equation (Figure 3). Physically, with the weakest frame, fluids have a maximum effect. It is interesting that for the Voigt bound, the fluid-saturation effect on the modulus increases with increasing porosity. This is opposite of the fluid-saturation effect on the modulus for the Reuss bound, which decreases with increasing porosity. At a porosity of 100%, both the Voigt and Reuss bounds in the Gassmann's calculation show that the fluid effect on modulus equals the fluid modulus.

In many reported applications, these general bound values have often been ignored. For example, a K_s calculated directly from log data (V_p, V_s, and bulk density) may be lower than the Reuss bound. This results in a negative value for K_d. Such bounds of the fluid effect on bulk modulus provide constraints for the input and output parameters of Gassmann's calculation.

Critical porosity

Reservoir rocks in general are far from the Voigt and Reuss bounds, as Figure 4 shows for sandstones. Dolomite with vuggy pores may approach the Voigt bound, and highly fractured rocks may approach the Reuss bound. However, there is a great difference between these idealized bounds and most real rocks: The bounds shown in Figure 4 do not limit most observed, naturally occurring porosity. The vast majority of rocks have an upper limit to their porosity, usually termed "critical porosity," ϕ_c (Yin, 1992; Nur et al., 1995). At this high-porosity limit, we reach the threshold of grain contacts (Han, 1986). This ϕ_c modifies the Voigt model (Figure 3) to provide tighter constraints for dry and fluid-saturated bulk moduli for sands. This triangle physically correlates with both the Voigt and Reuss bounds. The fluid-saturation effect on modulus is consistent with the Voigt triangle: It increase with increasing porosity and is limited by the Reuss bound at the critical porosity. The modified Voigt triangle provides a linear formulation and a graphic procedure for Gassmann's calculation: the fluid saturation effect on bulk modulus is proportional to normalized porosity and the maximum fluid saturation effect on bulk modulus (Reuss bound) at the critical porosity (Figure 3):

$$\Delta K_d = \phi / \phi_c \times K_{Rc}. \tag{19}$$

This is consistent with the earlier work done by Mavko and Mukerji (1995), with a slightly different physical formulation.

For typical sandstones, the critical porosity ϕ_c is around 40%. Thus, we can also generate a simplified numerical formula of the normalized modulus K_n for the modified Voigt model:

$$K_n(\phi) = 1 - \phi / \phi_c = 1 - 2.5 \times \phi. \tag{20}$$

Using this in Gassmann's equation (6) yields the fluid-saturation effect

$$\Delta K_d = \frac{6.25 \times \phi \times K_0}{1.5 + K_0/K_f} < 6.25 \times \phi \times K_f. \qquad (21)$$

EMPIRICAL MODEL FOR NORMALIZED MODULUS K_n

Extending our empirical approach to a first order, both P- and S-wave velocities can correlate linearly with porosity at high differential pressure ($P_d = 40$ MPa). For dry, clean sands (Han, 1986),

$$V_p = 5.97 - 7.85 \times \phi \text{ km/s},$$
$$V_s = 4.03 - 5.85 \times \phi \text{ km/s}, \qquad (22)$$

assuming that the density of these sands is equal to

$$\rho_d = 2.65 \times (1 - \phi) \text{ g/cm}^3. \qquad (23)$$

Because the modulus is the product of the density and the square of the velocity, we obtain an equation that is cubic in terms of porosity. The bulk modulus can be derived as

$$K_d = (1 - A \times \phi + B \times \phi^2 - C \times \phi^3) \times K_0, \qquad (24)$$

where $A = 3.206$; $B = 3.349$; $C = 1.143$. Equation (24) can be further simplified if porosity is not too high ($\phi < 30\%$):

$$K_d = (1 - D \times \phi)^2 \times K_0, \qquad (25)$$

where D for clean sandstone is equal to 1.52. This includes an empirical expression of the normalized modulus as a direct dependence on porosity and the "D" parameter. Table 1 and Figure 5 show empirical relations generated from dry velocity data of relatively clean rocks. The parameter D has been derived based on approximate linear velocity-porosity relations. D represents, in the first order, the correlation of porosity to bulk modulus for relatively clean sandstones and clastic sediments. It is related to rock texture, pressure, and fluid saturation and should be calibrated for local reservoir conditions. For shaly sands, the clay effect on the modulus should be corrected before the D parameter can be derived. For consolidated rock at high differential-pressure (>20 MPa) conditions, D ranges from about 1.45 to slightly more than 2.0, depending on consolidation of the rock.

By inserting this D function into equation (9) we find

$$G(\phi) = D^2 \times \phi \times (2 - D \times \phi)^2. \qquad (26)$$

Figure 6 shows the modified Voigt model based on critical porosity and the D function model, with $D = 2$. This simple calculation provides a useful quality-control tool to check any Gassmann-type calculation for sandstones.

SOLID MINERAL BULK MODULUS

As we mentioned previously, the normalized modulus K_n controls the fluid saturation effect, rather than K_0 or K_d individually. The mineral modulus K_0 is as important as is K_d. The K_0 is a mineral property. However, in most applications of Gassmann's equation, only K_d is measured. Properties of the mineral modulus K_0 are often poorly understood and oversimplified. K_0 is the modulus of the solid material that includes grains, cements, and pore fillings. If clays or other minerals are present with complicated distributions and structures, K_0 can vary over a wide range. Unfortunately, few measurements of K_0 have been made for sedimentary rocks (Coyner, 1984), and the moduli of clays are a particular problem (Wang et al., 1998; Katahara, 1996). Measured data (Coyner, 1984) show that at

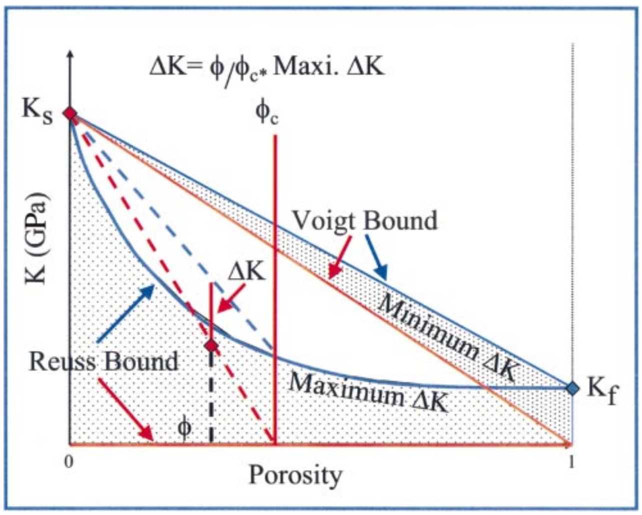

Figure 3. Illustration of both the Voigt high and Reuss low bounds for dry rock, fluid-saturated rock, and bounds of predicted fluid-saturation effect by the Gassmann's equation.

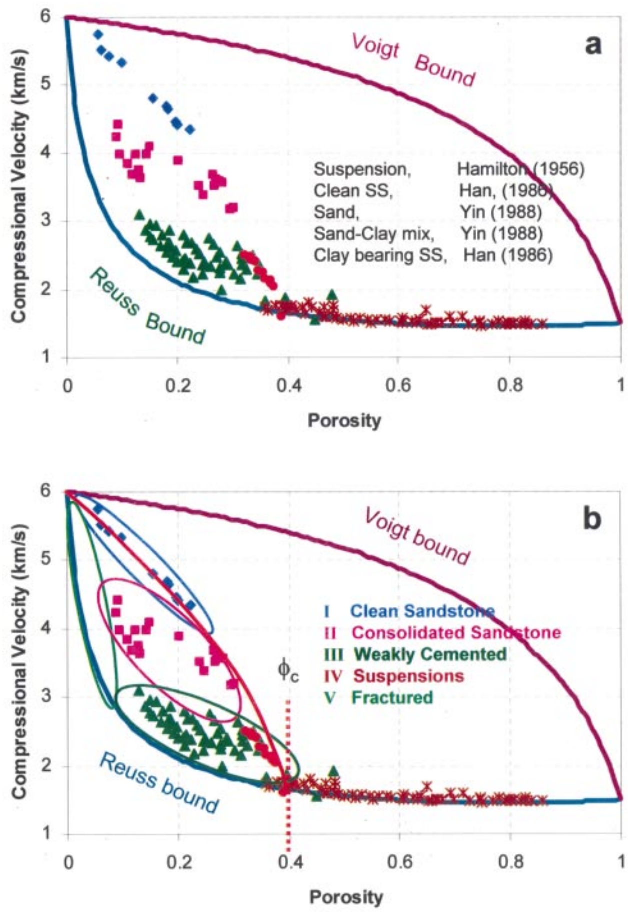

Figure 4. A typical velocity distribution for clean and shaly sands, weakly cemented sands, fractured rocks, and suspensions, in comparison with the Voigt and Reuss bounds and critical porosity (modified from Marion, 1990).

high pressures (>20 MPa), K_0 for sandstone samples ranges from 33 to 39 MPa. K_0 is not a constant and can increase by more than 10% with increasing effective pressure. Clearly, mineral modulus K_0 can vary across a wide range, depending on mineral composition, distribution, and in situ conditions.

Figure 7 shows the influence of K_0 on Gassmann's calculation. This case uses a dry bulk modulus calculated with a mineral modulus of 40 GPa and $D = 2$ in the D function. The water-saturation effect was calculated for three mineral moduli, of 65, 40, and 32 GPa, and a water modulus of 2.8 GPa.

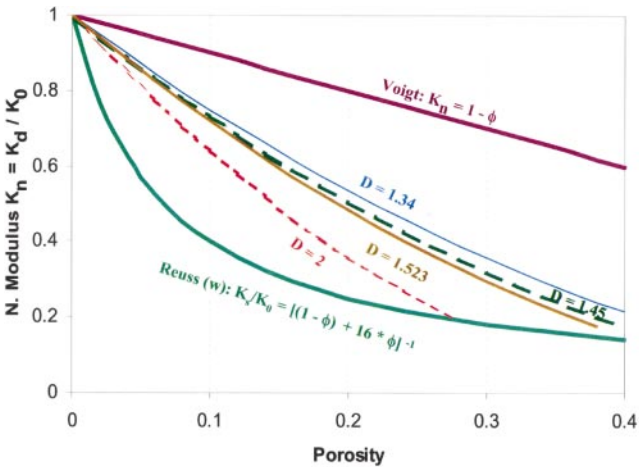

Figure 5. Normalized modulus based on Voigt and Reuss bounds and empirical D functions for different rocks (See Table 1).

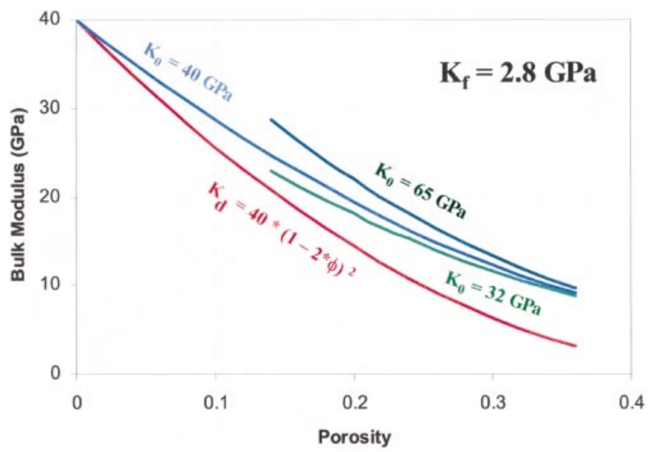

Figure 7. Different mineral-frame bulk-modulus effects on calculated bulk modulus with water saturation (brine modulus: $K_f = 2.8$ GPa) and $D = 2$ function model.

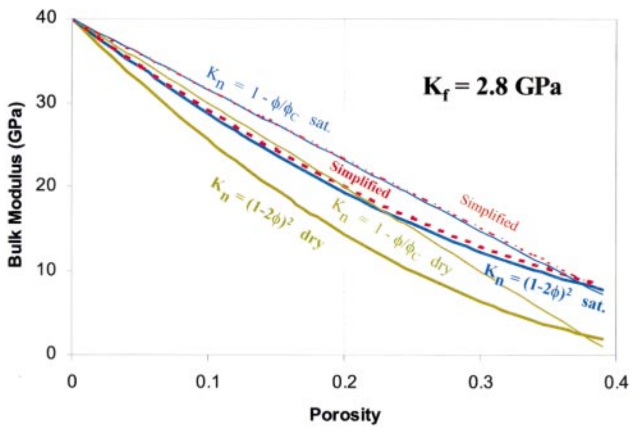

Figure 6. Comparison of the Gassmann and simplified Gassmann calculation for brine-saturation effect (brine modulus $K_f = 2.8$ GPa) for the modified Voigt bound, with $\varphi_c = 40\%$ and $D = 2$ function model.

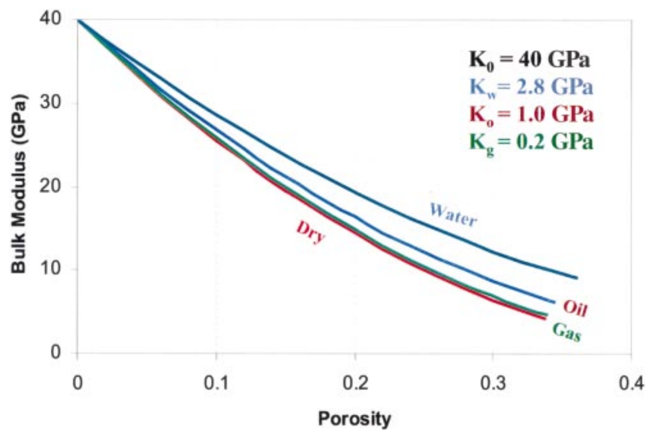

Figure 8. Fluid-saturation effect on calculated bulk modulus, with typical water, oil, and gas saturation and $D = 2$ function model.

Table 1. Compiled empirical relations and relative *D*-por. models for different rocks.*

Rock type	V-emp. relation $P_e = 40$ MPa	$K_d = (1 - A \times \phi + B \times \phi^2 - C \times \phi^3) \times K_0$			$K_d = (1 - D \times \phi)^2 \times K_0$
		A	B	C	D
Dry shaly sandstone	$V_p = 5.41 - 6.35 \times \phi$ $V_s = 3.57 - 4.57 \times \phi$	3.053	3.070	1.016	1.450 ($K_0 = 32.5$ GPa)
Dry clean sandstone	$V_p = 5.97 - 7.85 \times \phi$ $V_s = 4.03 - 5.85 \times \phi$	3.206	3.349	1.143	1.523 ($K_0 = 37.0$ GPa)
Silicate clastic (Castagna et al., 1985)	$V_p = 5.81 - 9.42 \times \phi$ $V_s = 3.89 - 7.07 \times \phi$	3.283	3.284	1.014	1.584 ($K_0 = 36.0$ GPa)
Dry vuggy limestone	$V_p = 6.47 - 5.84 \times \phi$ $V_s = 3.39 - 3.03 \times \phi$	2.815	2.639	0.824	1.340 ($K_0 = 71.9$ GPa)
Dry limestone	$V_p = 6.19 - 9.80 \times \phi$ $V_s = 3.20 - 4.90 \times \phi$	4.244	5.820	2.605	1.970 ($K_0 = 66.8$ GPa)
Dry dolomite	$V_p = 6.78 - 9.80 \times \phi$ $V_s = 3.72 - 5.20 \times \phi$	3.578	4.020	1.358	1.705 ($K_0 = 94.4$ GPa)

*K_d units are V-emp. relation.

Results show that for the same K_d and K_f, the bulk-modulus increment ΔK_d resulting from fluid saturation increases with increasing mineral modulus K_0. Errors resulting from uncertainty of K_0 can be significant for low-porosity rocks.

Because of a lack of measurements on mineral moduli, often we must use measured velocity-porosity-clay-content relationships for shaly sandstone (Han, 1986) to estimate the mineral modulus. Assuming zero porosity and a grain density of 2.65 g/cm^3, we can derive mineral bulk and shear moduli from measured P- and S-wave velocities. The results are shown in Table 2.

For relatively clean sandstones (with clay content of a few percent), the mineral bulk modulus (K_0) typically is 39 GPa, which is a stable value for differential pressures higher than 20 MPa. Mineral shear modulus (μ_0) is around 33 GPa, which is significantly less than the 44 GPa for a pure-quartz aggregate. Shear modulus is more sensitive to differential pressure and to clay content. For shaly sandstones, the mineral bulk modulus decreases about 1.7 GPa per 10% increment of clay content. Derived mineral bulk modulus can be used for the Gassmann's calculation, if there are no directly measured data or reliable models for calculation.

Lithology detection is often a goal of seismic interpretation. In modeling the seismic response, we often face the challenge of how to separate the fluid's influence from the lithology effect. To estimate this effect, we need to perform "lithology substitution" by using different values for the simplified gain function in Gassmann's equation:

$$\Delta K_{21}(\leq) \approx [G_2(\phi) - G_1(\phi)] \times K_f. \qquad (27)$$

We should incorporate this factor into the fluid-substitution scheme. However, this process is not commonly performed and is still poorly understood.

FLUID MODULUS AND FLUID-SATURATION EFFECTS

The fluid modulus is another key independent parameter in Gassmann's equation. Because identification of fluid types is often the primary goal of a seismic program, fluid properties are of critical significance. However, fluid properties are often oversimplified in seismic applications. Although complex, fluid properties are systematic. Oil properties depend on density (or API gravity), gas-oil ratio (GOR), gas gravity, pressure, and temperature conditions. Under different conditions, the fluid phase and its seismic properties can vary dramatically (Han and Batzle, 2000a, b; Batzle and Wang, 1992). As we mentioned previously, Gassmann's prediction is approximately proportional to the fluid bulk modulus K_f, with different gain functions. Thus, the stiffer the pore fluid, the higher the bulk modulus.

Table 2. Grain bulk and shear modulus for shaly sands; mineral modulus derived from empirical velocities relation of shaly sandstones. *C* is the fractional clay content.

	$C = 0$		$C = 0.1$		$C = 0.2$	
P_d (MPa)	K_0 (GPa)	μ_0 (GPa)	K_0 (GPa)	μ_0 (GPa)	K_0 (GPa)	μ_0 (GPa)
40	39.03	32.83	37.27	29.40	35.51	26.16
30	39.08	31.91	37.26	28.56	35.44	25.40
20	39.27	30.45	37.30	27.29	35.35	24.30
10	38.74	26.46	36.72	25.73	34.72	22.94

Fluid substitution is a primary application of Gassmann's equation. With a change of fluid saturation from fluid 1 to fluid 2, the bulk modulus increment is equal to

$$\Delta K_{21}(\leq) \approx G(\phi) \times (K_{f2} - K_{f1}), \qquad (28)$$

where K_{f1} and K_{f2} are the moduli of fluids 1 and 2, respectively, and ΔK_{21} represents the change in increment caused by substituting fluid 2 for fluid 1. Equation (28) uses the fact that the simplified gain function $G(\phi)$ of the dry-rock frame remains constant as the fluid modulus changes (this may not be true for some real rocks). The fluid-substitution effect on bulk modulus is simply proportional to the difference in fluid bulk moduli. This form of fluid substitution is similar to that derived by Mavko et al. (1998):

$$\frac{K_{s1}}{K_0 - K_{s1}} - \frac{K_{f1}}{\phi \times (K_0 - K_{f1})} = \frac{K_{s2}}{K_0 - K_{s2}} - \frac{K_{f2}}{\phi \times (K_0 - K_{f2})} = \frac{K_d}{K_0 - K_d}. \qquad (29)$$

Note that the fluid-substitution effect calculated through equation (29) is based on dry-frame properties. Deriving the dry modulus often helps us to examine validity of input parameters and output results of Gassmann's calculation.

If we know the simplified gain function for a rock formation, we can estimate the fluid-substitution effect without knowing shear modulus:

$$\rho_2 V_{p2}^2 \approx \rho_1 V_{p1}^2 + G(\phi) \times (K_{f2} - K_{f1}), \qquad (30)$$

where ρ_1, ρ_2, V_{p1} and V_{p2} are the densities and velocities of rock saturated, respectively, with fluid 1 and fluid 2. Both equations (28) and (30) are direct results from simplified Gassmann's equation (equation 8). Mavko et al. (1995) have suggested a similar method.

In Figure 8, we show the typical fluid-modulus effect on the saturated bulk modulus K_s. Even at a modest porosity of 15%, changes can be substantial. At in situ conditions, pore fluids are often multiphase mixtures. A dynamic fluid modulus may also depend on fluid mobility, fluid distribution, rock compressibility, and seismic wavelength.

Another approach is to use the P-wave modulus (M) to replace bulk modulus in Gassmann's equation. This works reasonably well for sandstones. The validity of this simplification results from the approximate equivalence of the ratio of dry-frame bulk and shear modulus (K_d/μ_d) to the ratio of mineral (quartz) bulk and shear modulus (K_0/μ_0).

ESTIMATING DRY BULK MODULUS

To estimate saturation effects on rocks practically, we need to obtain the dry bulk modulus. From Gassmann's equation, we derive K_d from fluid-saturated K_s and fluid-saturation effect ΔK_s:

$$K_d = K_s - \Delta K_s = K_s - \frac{K_0 \times (1 - K_s/K_0)^2}{\phi \times K_0/K_f + K_s/K_0 - 1 - \phi}, \qquad (31)$$

where the fluid-saturation effect ΔK_s is based on measured fluid-saturated modulus K_s and results in a slightly different formulation for ΔK_d in equation (2). If we reformulate ΔK_s

with the Reuss bound K_R of the rock with saturated fluid,

$$\Delta K_s = \frac{K_R \times (K_0 - K_s)^2}{K_0^2 + K_s \times K_R - 2 \times K_0 \times K_R} \le K_R. \quad (32)$$

Again, the Reuss bound provides the maximum fluid-saturation effect.

CONCLUSIONS

Gassmann's equations are used widely to calculate fluid-substitution effects. Unfortunately, the underlying assumptions are often violated, and the validity of the resulting calculations is unknown. Several factors can be incorporated into the analysis to make the results more physically meaningful and reliable.

1) A simplified form of Gassmann's equation clarifies the physical control of fluid-saturation effects on rock bulk modulus, through the simplified gain function of dry rock (dependent on the normalized dry bulk modulus and porosity) and fluid modulus. Rock parameters, such as normalized dry bulk modulus and porosity, are strongly correlated, which effectively reduces the number of free parameters.
2) The dry and fluid-saturated Voigt-Reuss bounds of bulk modulus provide physical limitations on Gassmann's equation. The minimum increment of bulk modulus resulting from fluid saturation, consistent with the Voigt bound, is proportional to porosity and fluid modulus. The maximum increment of bulk modulus with changing saturation, consistent with the Reuss bound, is equivalent to the Reuss bound of fluid-saturated rock itself. This is because the Reuss bound of dry rock is zero.
3) The normalized bulk modulus can be a complicated function of rock textures and in situ conditions, which may lead to wide variations in the results of applying Gassmann's equation. However, simplified modulus-porosity trends can be incorporated, resulting in simple polynomial dependence on porosity in sandstones. Fluid-substitution effects are then a straightforward function of porosity and the difference in fluid moduli.
4) The calculated dry-frame modulus or mineral modulus can be in substantial error if rock properties are not consistent or assumptions are violated.

Although these relationships have been applied for many years, considerable basic research still needs to be done on many of the controlling factors. Fundamental components, such as the mineral moduli (particularly for clays), are rarely measured. The exact character of the pore-volume modulus (Brown and Korringa, 1975) is unknown, and the validity of replacing it with the mineral modulus is ambiguous. The influence of mixed fluid phases and fluid mobility is also not incorporated. Additionally, theory applies strictly to the low-frequency range, thereby permitting no frequency dependence. With increasing application of seismic data to extracting and predicting reservoir and fluid properties, we will need more constrained and tested forms of Gassmann's relations and other porous-media theories.

ACKNOWLEDGMENTS

We thank the industry sponsors of "Phase III Fluid Consortium" for guidance and financial support. We also thank the editor and reviewers for their comments and suggestions.

REFERENCES

Batzle, M. L., and Wang, Z.-J., 1992, Seismic properties of pore fluids: Geophysics, **57**, 1396–1408.
Batzle, M. L., Han, D., and Castagna, J. P., 2001, Fluids and frequency dependent seismic velocities of rocks: The Leading Edge, **20**, 168–171.
Berryman, J. G., 1999, Origin of Gassmann's equations: Geophysics, **64**, 1627–1629.
Biot, M. A., 1941, General theory of three-dimensional consolidation: Journal of Applied Physics, **12**, 155–164.
Brown, J. S., and Korringa, J., 1975, On the elastic properties of a porous rock on the compressibility of the pore fluid: Geophysics, **40**, 608–616.
Gassmann, F., 1951, Über die elastizität poröser medien: Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich, **96**, 1–23.
Castagna, J. P., Batzle, M. L., and Eastwood, R. L., 1985, Relationship between compressional-wave and shear-wave velocities in clastic silicate rocks: Geophysics, **50**, 571–581.
Coyner, K. B., 1984, Effects of stress, pore pressure, and pore fluids on bulk strain, velocity and permeability in rocks: Ph.D. dissertation, Massachusetts Institute of Technology.
Hamilton, E. L., 1956, Low sound velocities in high porosity sediments: Journal of the Acoustical Society of America, **28**, 16–19.
Han, D., 1986, Effects of porosity and clay content on acoustic properties of sandstones and unconsolidated sediments: Ph.D. dissertation, Stanford University.
Han, D., 1992, Fluid saturation effect on rock velocities in seismic frequencies: presented at SEG Summer Research Workshop, Big Sky, Montana.
Han, D., and Batzle, M., 2000a, Velocity, density and modulus of hydrocarbon fluids—Data measurement: 70th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, paper RPB 7.2.
Han, D., and Batzle, M., 2000b, Velocity, density and modulus of hydrocarbon fluids—Empirical modeling: 70th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, paper RPB 7.3.
Katahara, K., 1996, Clay mineral elastic properties, Expanded abstract, 66th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, paper RP 1.4.
Marion, D. P., 1990, Acoustic, mechanical, and transport properties of sediments and granular materials: Ph.D. dissertation, Stanford University.
Mavko, G., and Mukerji, T., 1995, Pore space compressibility and Gassmann's relation, Geophysics: **60**, 1743–1749.
Mavko, G., Chan, C., and Mukerji, T., 1995, Fluid substitution: Estimating changes in V_p without knowing V_s: Geophysics, **60**, 1750–1755.
Mavko, G., Mukerji, T., and Dvorkin, J., 1998, Rock physics handbook: Tools for seismic analysis in porous media: Cambridge University Press.
Nolen-Hoeksema, R. C., 2000, Modulus-porosity relation, Gassmann's equations, and the low frequency elastic-wave response to fluids: Geophysics, **65**, 1355–1363.
Nur, A., Mavko, G., Dvorkin, J., and Gal, D., 1995, Critical porosity: The key to relating geophysical properties to porosity in rocks: 65th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, 878.
Sengupta, M., and Mavko, G., 1999, Sensitivity analysis of seismic fluid detection: 69th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, Paper BHRP 7.2.
Wang, Z.-J., Wang, H., and Cates, M. E., 1998, Elastic properties of solid clays: 68th Annual International Meeting, Society of Exploration Geophysicists, Expanded Abstracts, Paper RP 3.2.
Yin, H., 1988, Velocity in sand-clay mixture: Stanford Rock Physics Data.
Yin, H.-Z., 1992, Acoustic velocity and attenuation of rocks: isotropy, intrinsic anisotropy, and stress-induced anisotropy: Ph.D. dissertation, Stanford University.