Attenuation estimation with continuous wavelet transforms.

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SUMMARY

Seismic attenuation measurements from surface seismic data using spectral ratios are particularly sensitive to inaccurate spectral estimation. Spectral ratios of Fourier spectral estimates are subject to inaccuracies due to windowing effects, noise, and spectral nulls caused by interfering reflectors. We have found that spectral ratios obtained using continuous wavelet transforms as compared to Fourier ratios are more accurate, less subject to windowing problems, and more robust in the presence of noise.

INTRODUCTION

As a seismic wave propagates, it loses energy due to spherical divergence, scattering, and intrinsic attenuation (absorption). Spherical divergence is commonly assumed to be independent of frequency and a correction is commonly applied to seismic data that works reasonably well for simple geometries. Seismic scattering and attenuation may be related to both lithology and pore fluid content. It is well established that seismic waves passing through gas bearing strata exhibit anomalously high attenuation. This fact can potentially be used as a hydrocarbon indicator (Hedlin, 2001), especially if quantitative attenuation measurements can be made. The spectral ratio method is a popular means of measuring seismic attenuation. As ratios are particularly sensitive to noise and spectral errors, the precise estimation of the signal spectrum is a key to making robust attenuation measurements. The spectrum of the signal may be affected by many factors, for example, thin bed influences (Hackert et al 2004), interbed multiples and other noise, and windowing. We present a means to obtain improved spectral ratios through better estimation of the local signal spectrum using continuous wavelet transform (CWT) time-frequency analysis. In comparison to Fourier spectral estimation, the CWT method exhibits several advantages which we will investigate in this paper.

It is common practice to express seismic attenuation in terms of the quality factor (Q). The spectral amplitude-ratio technique is a popular method of estimating Q, because it is independent of the source influence. Accurate estimation of the signal spectrum is a key to accurately determining Q. However, in practice there are many difficulties to estimate the signal spectrum correctly. First, it is almost impossible to remove the reflectivity spectrum from the spectral estimates. This would require isolating and performing spectral estimation on reflections from two individual reflectors. However a seismic event is usually is a complex superposition of many reflections. Second, the conventional spectral analysis method using Fourier Transforms temporally localized by windowing distorts the spectrum. Third, the noise spectrum will certainly cause severe degradation in the low SNR frequency range (Turner and Siggnis 1994).

Wavelet time-frequency analysis provides a new approach to determining the local characteristics of a signal. A variety of wavelet analysis methods had been applied in the literature for estimating Q since Dr. Taner proposed doing so in 1983. James and Knight (2003) applied the S-transform to measure the centroid frequency shift to calculate Q for ground penetrating radar data. This method can be applied to seismic reflection data in an analogous fashion. In this paper, we discuss the estimation of instantaneous spectra of the signal using the continuous wavelet transform (CWT) and compute Q in the time-frequency domain using spectral-ratios. We find that this method exhibits several advantages: (1) In comparison to conventional Fourier spectral estimation with wide windows, the CWT can directly determine the signal spectrum for individual events. These events may be composite signals of sub-resolution reflectors, but the severe spectral notching that characterizes wide window Fourier spectral estimation from multiple events in the window is reduced, (2) The CWT avoids smearing the spectrum as a consequence of the temporal window influence, and (3) Spectral ratios derived from CWT spectra are more robust in the presence of noise.

In this paper we study the effects of windowing and noise in comparing Fourier and CWT attenuation estimation on both synthetic seismograms and laboratory signals.

METHOD

By definition, attenuation affects the amplitude spectrum of the propagating seismic wavelet and thus the resulting reflection seismogram. A spherical harmonic wave A(R,ω)propagating in an attenuating medium can be described as:

$$A(R,\omega) = A_0(R_0,\omega)G(R)G(I)K(r)\exp(-\alpha(\omega)R)\exp(-i\omega R/V) \quad (1)$$

where $\alpha(\omega)=\omega/(2VQ)$, A(R, ω) is the wave amplitude at a distance R from the source, $A_0(R_0,\omega)$ is the amplitude at the source, V is the frequency dependent wave velocity in the medium, G(R) is geometric spreading, G(I) is instrument response, and K(r) is the loss due to reflection and transmission. The exponential term exp (-α(ω)R) is the anelastic attenuation in the medium. The second exponential term $\exp(-i\omega R/V)$ is a phase delay. If we assume that the attenuation coefficient α(ω) is linearly dependent upon frequency within a limited bandwidth then Q is nearly constant over the frequency range providing the velocity dispersion is small. In reality, a seismic record contains a superposition of reflections from a great many impedance contrasts that will overlap temporally with one another. However, suppose that we can isolate two discrete seismic events. Then the spectral ratio method for estimating Q can be expressed as follows:

$$\frac{A_2(R_2,\omega)}{A_1(R_1,\omega)} = \frac{R_1}{R_2}\cdot\frac{G(I_2)}{G(I_1)}\cdot\frac{K(r_2)}{K(r_1)}.\exp\alpha(\omega)(R_1-R_2).\exp(i\omega(R_1-R_2)) \quad (2)$$

The exponential term $\exp(i\omega(R_1-R_2))$ is purely a time delay that does not enter into the amplitudes, and therefore it can be omitted. Then we obtain:

$$\alpha(\omega) = (R_1-R_2)^{-1}\{\log(\frac{A_2(R_2,\omega)}{A_1(R_1,\omega)}) - \log(\frac{R_1}{R_2}) - \log(\frac{G(I_2)}{G(I_1)}) - \log(\frac{K(r_2)}{K(r_1)})\} = \frac{\pi f}{QV} \quad (3)$$

$$\log(\frac{A_2(R_2,\omega)}{A_1(R_1,\omega)}) = \frac{\pi f}{QV}(R_1-R_2) + const \quad (4)$$

Here ,

$$const = \log(\frac{R_1}{R_2}) + \log(\frac{G(I_2)}{G(I_1)}) + \log(\frac{K(r_2)}{K(r_1)})$$

Thus the logarithm of the ratio of the spectra of the two reflected wavelets is assumed to be a linear function of frequency whose slope will estimate Q. In particular, if reflection K_{th} is from the top of an interval containing a gas reservoir and reflection J_{th} is from the bottom of that interval, then we might hope by this method to estimate the Q value of the interval containing the reservoir.

CONTINOUS WAVELET TRANSFORM

CWT is a time-frequency analysis method that is essentially narrow band filtering of a time series with a wavelet dictionary. A wide selection of wavelets can be employed as long as they satisfy predefined mathematical criteria (Gonzales, 2002). We implement the wavelet transform by computing a convolution of the seismic trace with a scaled wavelet dictionary. We need to convert the scale dependent wavelet energy spectrum of the signal, E(s), to a frequency dependent wavelet energy spectrum in order to compare directly with the Fourier energy spectrum of the signal. The different wavelets we choice may influence the time and frequency resolution. For example, the Morlet wavelet is very well localized in the frequency domain. The Morlet wavelet response pairs in the time and frequency domains are:

$$g_j(t) = \exp(-a_j t^2)\exp(iw_j t) \quad (5)$$

$$G_j(w) = \int_{-\infty}^{\infty} g_j(t)\exp(iwt)dt = \sqrt{\frac{\pi}{a_j}}.\exp(\frac{-(w-w_j)^2}{4a_j}) \quad (6)$$

Where a_j is scale. We may choose $a_j = \frac{\ln 2}{4\pi^2}\omega_j^2$ and design narrow band filters (see Figure 1) that constitute the wavelet dictionary.

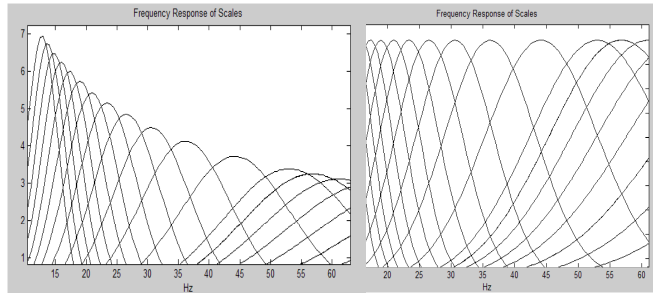

Figure 1. A Morlet wavelet dictionary in the frequency domain (left) and its normalization (right).

We use a synthetic seismic signal to study the localization properties of CWT methods. Figure (2a) shows a synthetic trace produced using Ricker wavelets with variable center frequency as source wavelets. The synthetic trace is a superposition of the wavelets. Figure (2b) is the energy spectrum distribution in the time-scale (frequency) domain; Figure (2c) results from the application of a scale filter in the time-scale domain. In this figure, the signal can be distinguished clearly.

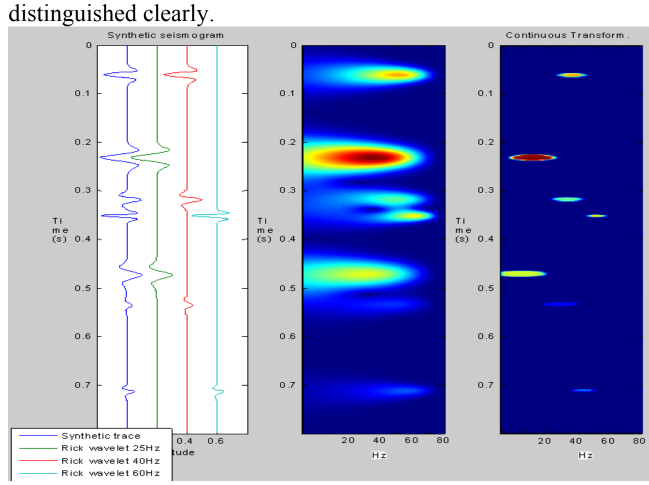

Figure 2. (a) synthetic seismic trace. (b) time-frequency decomposition. (c) The result of (b) with threshold window filter.

SYNTHETIC EXAMPLE AND APPLICATION

In Q computation, we need to compute the amplitude spectral ratio of two events as shown synthetically in Figure 3. We used a 25 Hz Ricker wavelet as a source wavelet, the wave velocity is 3000m/s, the distance between two events is 1100m, and the constant Q is 40. Figure 4 shows the effect of added Gaussian noise (SNR=12 dB). The Fourier amplitude spectrum (Figure 4a) shows the spectral oscillation caused by noise, which influences the slope of a best-fit line and the accuracy of Q estimation, However the CWT time-frequency plot shows less oscillation due to noise (Figures 5c and 5,e).

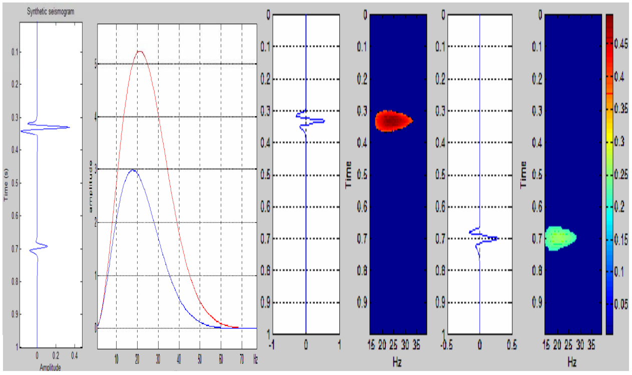

Figure 3. (a) and (b) are two synthetic events and their amplitude spectra, the red line is the first event and blue line is second event. (cdef) are signals and corresponding time-frequency decompositions with CWT.

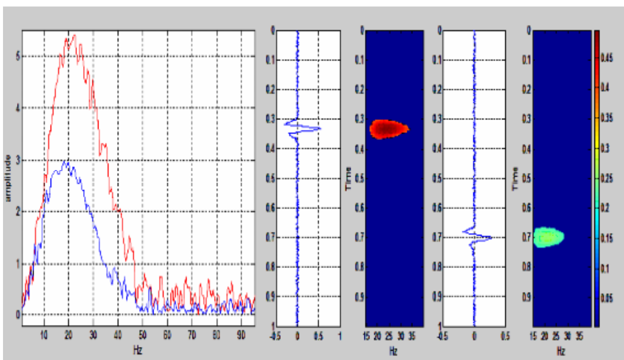

Figure 4. (a) are the amplitude spectra of two synthetic events with noise, red line is the first event and blue line is second event. (bcde) are signals and their time-frequency decompositions with CWT.

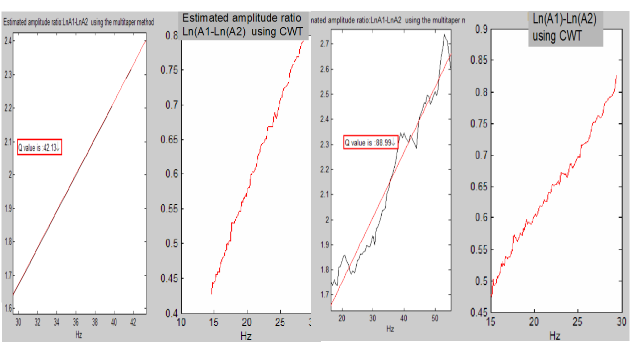

Figure 5. (a) is the noise free Fourier amplitude spectral ratio compared to the correct spectral ratio. (b) is the amplitude spectral ratio line generated by CWT. (c,d) correspond to (a) and (b) with Gaussian random noise added.

Since seismic data is band limited, our calculation of amplitude spectral ratio is located within the band of the data. If we select the frequency range outside the range of the signal, the results of estimated Q value would be unreliable. The two amplitude spectral ratio lines generated by windowed FFT and CWT respectively are shown on the Figure 5. Without adding noise to signal, the Q values determined by using equation (3) to be 42.13 using the FFT and 39.8 using the CWT which is very close to the true value of Q=40. But, with noise, the FFT gives Q=88.99 while the CWT spectral ratio gives Q=38.4. The large error using Fourier spectra comes from the interplay of windowing and noise [7].

We also applied this method to estimate the Q values for ultrasonic pulse transmission measurements of a sandstone sample. The rock sample density is 2.16 g/cc, porosity is 31.91%, water saturation is 100%, pore pressure is 500psi and confining pressure is 3500psi. Figure (6a) is the data measured in laboratory. The waveform A is the reflected waves from the top of the sample and C is the transmitted wave. The schematic diagram of the measurement is shown in Figure (6b).

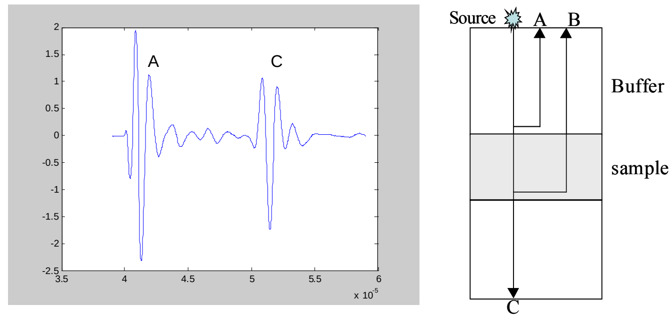

Figure 6. (a) The wave A is reflected from the top of the sample and C is the transmitted wave. (b) The schematic diagram of the measurement.

Fourier amplitude spectral of the transmitted wave shows that its amplitude is greater than that of the reflected wave around 20kHz-50kHz, and its amplitude dramatically changes at 70kHz (Figure 7a). This phenomenon shows that the transmitted wave is not a pure transmitted wave and it may be a superposition of a transmitted wave and a reflected wave from the side edge (Figure 7c and 7d). Estimating a Q value using Fourier spectra will be influenced by the notching, which was produced by two such closely spaced arrivals. For example, Q

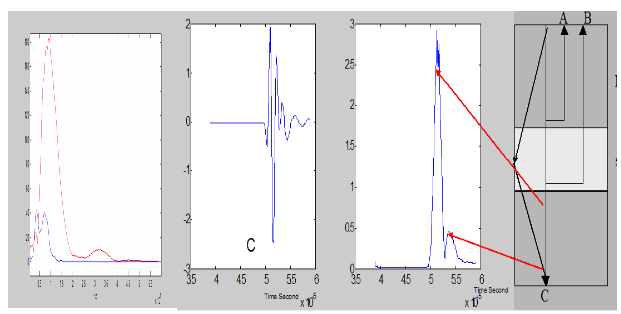

Figure 7. (a) is the amplitude spectrum of the signal (FFT), Red and blue lines are A wave and C wave, respectively. (c) is the amplitude spectrum of Hilbert transform, note the second peak of amplitude. (d) is the diagram of the possible path of C wave.

will be determined to be negative in the 20-50 kHz frequency range in the case. In contrast, the CWT method may avoid the temporal window influence and directly consider two local events. Figure (8a) is CWT time-frequency spectrum of the signal, and Figure (8b) is the threshold window filter applied to time frequency spectrum in order to focus local energy and better show the downward trend in the dominant frequency of the signal with time. In the time-frequency plane, a clear downward shift in the dominant frequency of the trace with

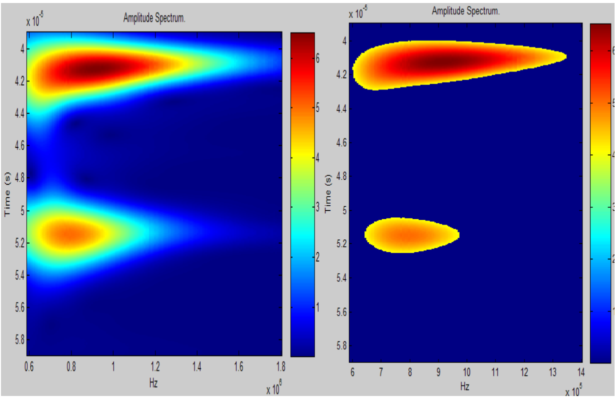

Figure 8. (a) CWT time-frequency spectrum of the signal, (b) the threshold window filter applied to (a).

time is evident as a result of wavelet attenuation. The dominant frequency drops from 0.92 MHz to approximately 0.78 MHz over the interval shown. Using our Q estimation procedure computed Q=45.6. To examine the accuracy of this Q value, we applied it to forward modeling through taking the first waveform A as a source signal and letting it propagate to the C location with Q=45.6. Figure 9 shows that the forward modeled signal significantly fits the original signal Q in dominant frequency and there is only a minor discrepancy in the relative amplitudes of the peaks.

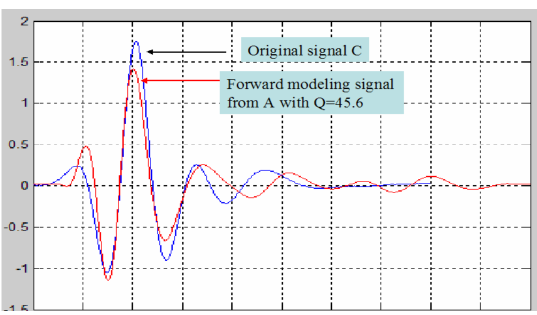

Figure 9. The blue line is the transmission wave C. red line is the wave generated by the reflector wave A with forward modeling.

CONCLUSIONS

It has been shown that the continuous wavelet transform is a flexible time-frequency decomposition tool that can form the basis of useful signal analysis and coding strategies. This spectral analysis method makes it possible to estimate attenuation Q value directly between two events. In comparison to Fourier spectral ratio for Q estimation, the CWT time-frequency spectral decomposition appears to provide a more robust and effective means of estimating Q.

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